

# A method for the design of a scaled wind turbine for wind tunnel experiments

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The current work introduces a blade element momentum theory-based tool called BEMTurbine for designing and evaluating the performance of wind turbines. BEMTurbine is a python-based open source tool that can be used to optimize blade parameters (chord length and twist angle) for the aerodynamic performance. This tool is then used to design a model wind turbine with a rotor diameter 0.25 m and the optimum tip speed ratio of 5. Rotor of this turbine is manufactured using a 3D printer. The BEM analysis shows that the maximum power coefficient of the model turbine is 0.47, and is attained at the design tip speed ratio of 5.

Keywords: Wind Turbine, Blade Element Momentum Theory, Wind Tunnel Experiment

## 1. Introduction

Wind energy researches rely on (1) field measurements, (2) numerical simulations and (3) wind tunnel experiments, in order to improve our understanding of the physics governing the wind energy systems and ultimately to improve their design and performance. While field measurements with full scale wind turbines can sound most reliable among the three approaches, they pose severe limitations in terms of what can be measured and can be extremely expensive. Because the environmental conditions cannot be controlled, it is not always possible to perform desired measurements for the desired length of time. Numerical simulations are viable alternatives, and in particular computational fluid dynamics (CFD) have shed light on wind turbine wake dynamics and interaction of wind turbines (or wind farms) with the atmospheric boundary layer (ABL) <sup>1,2</sup>. However, it is still prohibitively expensive to fully resolve wind turbines in CFD simulations. Therefore, most investigations of wind turbines and farms which are based on large-eddy simulation (LES), employ actuator-disk or actuator line models <sup>3</sup>, to impose turbine-induced forces on the flow. These models however, require validation and calibration of their model parameters. This brings us to the third approach, i.e., scaled wind tunnel experiments which are used to parameterize and validate numerical models. Experiments also help to measure valuable data—in a controlled environment—which for financial or technical

reason cannot be measured through field tests. Note that Reynolds number (and thus flow scales) of actual ABL are larger by two to three order of magnitude than what can be attained in wind tunnel experiments. Therefore, experiments cannot reproduce all relevant physics. Nevertheless, they provide valuable complement to the field measurements and numerical simulations <sup>4</sup>.

One of the most detail scaled model of a wind turbine for wind tunnel experiments has been described by Bottasso *et al.* <sup>5</sup>. In addition to aerodynamics, their model has also been used for testing aeroelastic properties and several non-conventional controls. Schottler *et al.* <sup>4</sup> used their controllable model wind turbine to investigate the effect of turbulent, intermittent inflow and control action on the torque fluctuation experienced by the turbine. Stein *et al.* <sup>6</sup> used a model wind turbine of 0.45 m rotor diameter to investigate the evolution of mean wind speed and turbulence intensity in wind turbine wake for a range of tip-speed ratio.

The studies reviewed above and many other works in the literature (see e.g. Ref <sup>7</sup>), design and manufacture their experimental wind turbine based on the requirement of the investigation. However, they do not provide sufficient details regarding the design process and the basis of selecting major design parameters. The current work presents a blade element momentum (BEM) theory-based tool for optimizing wind turbine rotor blades and evaluating the performance of the final design. A scaled model wind turbine designed using this

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tool is then introduced.

## 2. BEM-based tool development

This section presents method for estimating blade design and calculating aerodynamic performance of wind turbine. To this end, blade element momentum (BEM) theory is used.

### 2.1. Theoretical background

BEM theory is the combination of momentum theory and blade element theory. Discussion of BEM theory presented over here is based on Manwell *et al*<sup>8</sup>. For the analysis, blades are divided into elements (or section) and it is assumed that there is no aerodynamic interactions between these elements. Figure 1 illustrate the schematic of BEM analysis. From the conservation of linear and angular momentum in the control volume through the blade element-swept annular ring, the differential thrust ( $dT$ ) and the differential torque ( $dQ$ ) can be expressed as:

$$dT = \rho U^2 4a(1-a)\pi r dr, \quad (1)$$

$$dQ = 4a'(1-a)\rho U \Omega r^3 \pi dr. \quad (2)$$

Here,  $U$  is freestream inflow velocity,  $\rho$  is air density,  $r$  is radial distance of the element from the rotor center,  $dr$  is thickness of the element,  $\Omega$  is angular velocity of the rotor,  $a$  is axial induction factor,  $a'$  is angular induction factor.

The thrust and torque on blade elements can also be determined from lift and drag characteristic of the airfoil to be used in each blade element. The angle of relative wind ( $\phi$ ) experienced by an element is the sum of twist angle ( $\theta_p$ ) and the angle of attack ( $\alpha$ ) for that element,

$$\phi = \theta_p + \alpha. \quad (3)$$

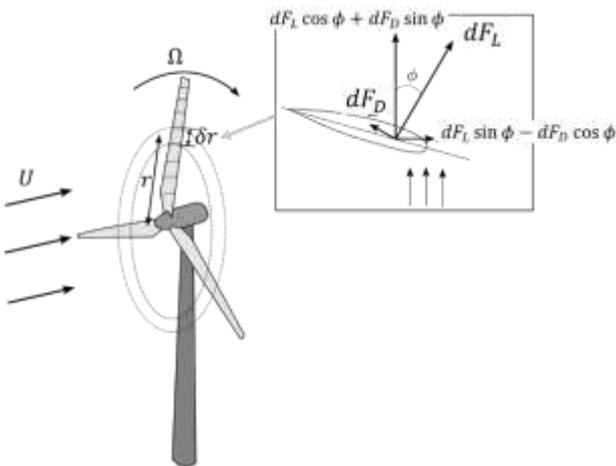


Figure 1. Wind turbine blade divided into elements. Force components on an element is shown using airfoil cross section.

Relative velocity is defined as:

$$U_{rel} = \sqrt{U^2(1-a)^2 + \Omega^2 r^2(1+a')^2} \quad (4)$$

Without going into the mathematical details, following expressions relating thrust and torque to the lift and drag forces can be determined:

$$dT = dF_L \cos \phi + dF_D \sin \phi \\ = \frac{1}{2} \rho U_{rel}^2 B c (C_L \cos \phi + C_D \sin \phi) dr \quad (5)$$

$$dQ = (dF_L \sin \phi - dF_D \cos \phi) r \\ = \frac{1}{2} \rho U_{rel}^2 B c r (C_L \sin \phi - C_D \cos \phi) dr \quad (6)$$

Here,  $B$  is number of rotor blades,  $c$  is chord length,  $C_L$  and  $C_D$  are lift and drag coefficients respectively. Equating Eq. (1) with (5) and Eq. (2) with (6),

$$\frac{U_{rel}^2}{U^2} B \frac{c}{R} (C_L \cos \phi + C_D \sin \phi) dr = 8\pi a(1-a)\mu, \quad (7)$$

$$\frac{U_{rel}^2}{U^2} B \frac{c}{R} (C_L \sin \phi - C_D \cos \phi) = 8\pi a'(1-a)\mu^2 \lambda, \quad (8)$$

where  $\mu = r/R$  and tip speed ratio  $\lambda = \Omega R/U$ . Note that  $\phi$  is also given by,

$$\tan \phi = \frac{U(1-a)}{\Omega r(1+a')} = \frac{(1-a)}{(1+a')\lambda\mu}. \quad (9)$$

Eq. (7) and (8) should be solved iteratively to obtain values for flow induction factors  $a$  and  $a'$ . These induction factors computed for all the blade elements are substituted in Eq. (1) and (2) and integrated over the rotor to compute total thrust force and torque of the turbine. If torque is known, power output is given by:

$$P = Q\Omega. \quad (10)$$

BEM theory is also used to optimize the blade shape by assuming ideal turbine operation. For maximum power output, axial induction factor  $a = 1/3$ <sup>8</sup>. Other assumptions related to idealized rotor are, there is no wake rotation, i.e.,  $a' = 0$ ; there is no tip loss. First, a design tip speed ratio ( $\lambda$ ), number of blades, rotor diameter, airfoil type for each blade section are chosen. For each element, the angle of attacks for which ( $C_D/C_L$ ) are minimum in the airfoil tables are selected.  $C_L$  values at these angle of attacks are used in the blade design. This allows to assume that  $C_D$  is negligible, which is one of the condition for blade optimization. Substituting these assumptions in Eq. (7) and (9) and after some rearrangements, one can obtain following optimum values for each blade element:

$$\phi = \tan^{-1} \left( \frac{2}{3\lambda\mu} \right), \quad (11)$$

$$c = \frac{8\pi r \sin \phi}{3B C_L \lambda \mu}. \quad (12)$$

$\phi$  from Eq. (11) can be substituted into Eq. (3) to obtain local twist angle. Chord length and twist angle distributions thus obtained are used to design optimum

rotor.

## 2.2 BEMTurbine

This section briefly introduces BEMTurbine <sup>9</sup>, an open source tool for optimizing aerodynamics design of wind turbine rotors and for evaluating their performance. The tool is based on BEM theory discussed above. This is a python-based tool, and has two main functions called *optimum\_rotor* and *BEM\_analysis*. The former optimizes the blade shape (twist and chord), while the later compute power and thrust coefficients as a function of tip speed ratio. Users do not need to go deep inside these functions, instead they can simply define all necessary parameters in the input files inside setup folders. The names of the input files correspond to each of the functions.

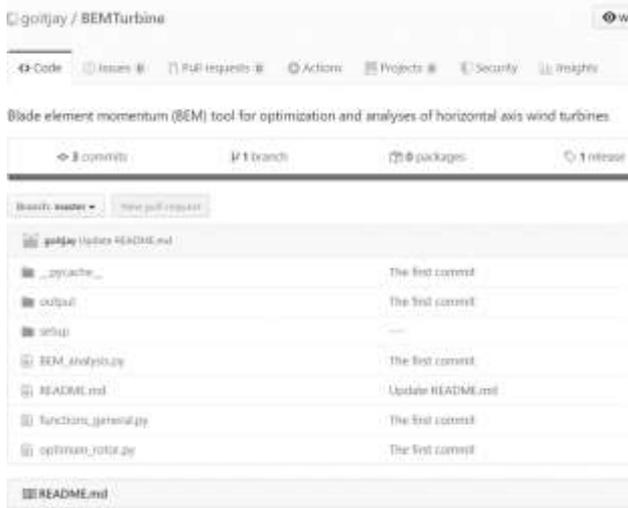


Figure 2. BEMTurbine on github <sup>9</sup>.

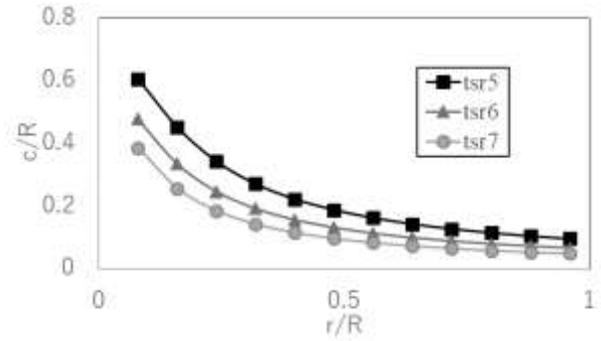
Some main input parameters required in *BEM\_analysis.inp* are rotor diameter, number of blades, tip speed ratio range for analysis, number of blade elements and airfoil properties, twist angle, section thickness for each of the elements. Similarly, main input parameters of *optimum\_rotor.inp* are rotor diameter, number of blade, design tip speed ratio, number of elements and airfoil properties for each of the elements.

## 3. Scaled wind turbine design

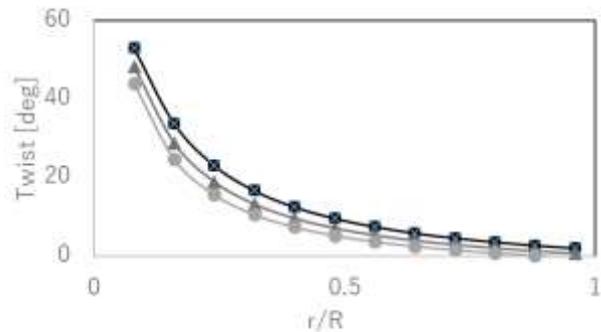
This section describes the design of a scaled wind turbine using the BEMTurbine. Rotor diameter,  $D = 0.25$  m is selected so that the model turbine can be comfortably installed—without significant blockage effect—in wind tunnel test section of 1.5 m to 2.0 m height. This height range corresponds to the test section heights of commonly available wind tunnel for mechanical and civil engineering experiments. Another goal is to test interactions between turbines in an array

which requires separation of 4D to 7D between turbines. To perform such experiments even in sufficiently large wind tunnel, requires that rotor diameter is not too large.

Following earlier studies, SD7003 airfoil profile is used along the entire blade <sup>4,6</sup>. The current design uses  $C_L$  and  $C_D$  profiles at Reynolds number of 50,000. At this Reynolds number, the minimum  $C_D/C_L = 0.0308$  occurs at angle of attack,  $\alpha = 6^\circ$ . For  $\alpha = 6^\circ$ ,  $C_L$  and  $C_D$  are 0.789 and 0.0244 respectively.



(a) Non-dimensional blade chord



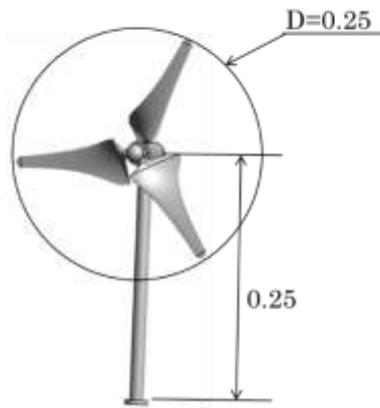
(b) Twist angle

Figure 3. Blade chord and twist angle as a function of radial distance from the rotor center. R is the radius of the rotor.

Figure 3 shows chord length and twist angle as a function of radial location on the blade, computed using BEMTurbine. The parameters are optimized for three different tip speed ratio, i.e.,  $\lambda = 5, 6$  and 7. Both chord length and twist angle increase towards the rotor center. They also increase with decreasing design tsr values. Most utility scale wind turbines are designed for optimum tsr of 7. To ensure that the model wind turbine has similar aerodynamic characteristics as utility scale wind turbines, it should also be designed for the same tsr value. However, a small model wind turbine with higher tsr, has to operate at a very high rotationla speed. Due to larger contribution of friction loss from motor and other moving components, it is not practical to attain such a high rotor speed. The consequence is that

such turbine show significant poor performance. As a compromise, the model in the current work is designed for the optimum trs of 5.

Figure 4 (a) shows the final design of the wind turbine, including rotor, nacelle and tower. As stated earlier rotor diameter is 0.25 m. Nacelle is designed to fit a small DC generator of diameter 0.02 m. The height of the tower is chosen such that the hub height is same as the rotor diameter, i.e., 0.25 m. Figure 4 (b) shows the 3D printed rotor. The material used in 3D printing is Veroclear (RGD810). It has a tensile strength between 50 to 65 MPa which is higher than the tensile strength of ABS, the commonly used plastic material for 3D printing. However, because the blades are very thin near the tip, they are flexible and may not withstand force due to wind during experiments.



(a) Complete wind turbine design



(b)

Figure 4. Designed and fabricated wind turbine.

Performance of the model wind turbine is analyzed by evaluating its power coefficient ( $C_p$ ) and thrust coefficient ( $C_T$ ) which are defined as:

$$C_p = \frac{P}{\frac{1}{2}\rho U^3 A_t}, \quad (13)$$

$$C_T = \frac{T}{\frac{1}{2}\rho U^2 A_t}. \quad (14)$$

where  $A_t$  is the area of the turbine rotor. Figure 5 shows  $C_p$  and  $C_T$  as a function of trs. Both coefficients initially increase with trs and reaches the maximum value at the optimum trs of 5 and beyond that they

decrease. The maximum value of  $C_p$  is 0.47 and the maximum  $C_T$  is 0.876. This  $C_p$  value is very high and compares with the maximum  $C_p$  of utility scale wind turbines. The power coefficient of idealized Betz turbine<sup>8</sup> without any losses is 0.5926. Therefore, performance of the model turbine designed in this study can be considered to be good. However,  $C_p = 0.47$  was computed from the BEM analysis and not from actual measurements. In the next step, performance of the turbine will be evaluated through wind tunnel experiments.

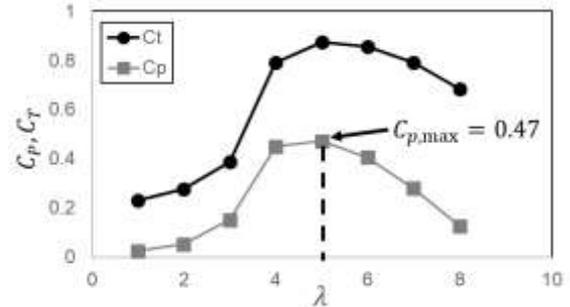


Figure 5. Power and thrust coefficient as a function of tip speed ratio.

#### 4. Summary

The current work has introduced a BEM theory-based tool called BEMTurbine for design and performance analysis of wind turbines. The tool has then been used for aerodynamic design of a model wind turbine for wind tunnel experiments. The analysis showed that the maximum power coefficient of the model wind turbine was 0.47, which was comparable to the performance of utility scale wind turbines. The next step will be to evaluate aerodynamic and elastodynamic performances of the turbine via wind tunnel experiments.

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